INSURANCE EXAM · IC-28

IC-28 Foundation of Actuarial Science

Revision book

15	2	243
chapters	master tables	numbered sections

FREE SAMPLE: Chapter 1 of 15

CHAPTER 1

Compound Interest, Accumulated Value, Present Value, Investment Returns

1.1 Loan words
1.2 Simple v compound
1.3 Tables
1.4 Present value
1.5 Nominal and effective
1.6 Discount
1.7 Varying rates

MUST KNOW

1. **Simple interest:** A = C (1 + n i), interest = C n i. Unpaid interest earns nothing.
2. **Compound interest:** A(n) = C (1+i)^n, interest = C [(1+i)^n - 1]; holds for fractional n too.
3. Present value of 1 due in n years = v^n = (1+i)^-n, v = 1/(1+i). PV is always less than the sum due.
4. **Effective from nominal:** i = (1 + i(m)/m)^m - 1. Nominal from effective: i(m) = m [(1+i)^(1/m) - 1].
5. d = 1 - v = i/(1+i) = i v. i = d/(1-d). i - d = i d. v^n = (1-d)^n.
6. **Varying rates:** multiply stage factors (1+i)^m (1+i')^n (1+i'')^p.
7. **Rs 100 at 10% for 3 yrs:** simple **Rs 130**; compound or yearly reinvestment **Rs 133.10**.

1.1 Loan words

- Borrower uses capital; lender or investor allows it.
- Principal = amount borrowed; time = period; interest = extra paid; amount = principal + interest.
- Default = loss of capital, interest or both. Higher default risk = higher expected rate.
- p.a. = per annum.

TRAP

Higher risk of default means a HIGHER rate, not lower.

1.2 Simple v compound

- **Simple:** unpaid interest does not earn interest.
- **Compound:** unpaid interest is an extra loan for the next period at the same rate.

FORMULA AND WORKED SUM

- **Simple:** A = C (1 + n i); interest = C n i.
- **Compound:** A(1) = C(1+i); A(2) = C(1+i)^2; A(n) = C(1+i)^n.
- **Rs 100 at 10%, 3 yrs:** simple 100 x 1.3 = Rs 130 (interest 30).
- **Yearly reinvest at 10% simple:** 110, 121, 133.10 (interest 33.10; Rs 3.10 more).
- Rs 2,000 at 6% simple 4 yrs = Rs 2,480. Rs 5,000 at 8% compound 3 yrs = Rs 6,298.56.

TRAP

- Extra **Rs 3.10** comes from interest on interest, not a higher rate.
- Compound formula is not limited to whole n.

1.3 Tables

- **Tables of (1+i)^n:** rates up to **20%**, n from 1 to 50. Tables of v^n also given.

FORMULA AND WORKED SUM

- **Rs 250, 10 yrs, 6%:** 250 x 1.79085 = Rs 447.71; interest Rs 197.71.
- **Amount Rs 3,969, 5%, 2 yrs:** C = 3,969 / 1.1025 = Rs 3,600.
- **Amounts Rs 8,820 (2 yrs) and Rs 9,261 (3 yrs):** ratio 1.05, i = 5%, C = Rs 8,000.

TRAP

Ratio of two neighbouring years' amounts = 1+i.

1.4 Present value

- v = 1/(1+i); v is "Value". PV of 1 due in n years = v^n.
- Finding PV = discounting; PV = discounted value.
- **v^n does not show i:** state the rate separately.

FORMULA AND WORKED SUM

- **Rs 300 due 5 yrs at 6%:** 300 x 0.74726 = Rs 224.18.
- **Rs 200 (3 yrs) + Rs 400 (5 yrs) at 5%:** 172.768 + 313.412 = Rs 486.18.
- **Rs 500 due 4 1/4 yrs at 7%:** 500 x 0.76290 x 0.98323 = Rs 375.05.
- **Rs 1,000 due 4 yrs at 8%:** 1,000 / 1.08^4 = Rs 735.03.
- **Rs 300 (2 yrs) + Rs 600 (4 yrs) at 10%:** 247.93 + 409.81 = Rs 657.74.

TRAP

Later payment = smaller present value.

1.5 Nominal and effective

- Period can be a year, half-year, quarter or month.
- **4%** p.a. quarterly = **1%** per quarter; 5 1/2 yrs = 22 quarters.

FORMULA AND WORKED SUM

- **Rs 200, 5 1/2 yrs, 4% quarterly:** 200 x 1.01^22 = Rs 248.944.
- **Rs 100 at 4%:** yearly Rs 104; half-yearly 102 + 2.04 = Rs 104.04.

- More frequent conversion = more interest in a given time.
- Effective rate i = actual yearly interest on 1. Nominal i(m) = m times the rate per period.

FORMULA AND WORKED SUM

- **8% convertible quarterly:** 1.02^4 - 1 = 8.243% effective.
- **8% effective:** i(4) = 4 [1.08^(1/4) - 1] = about 7.77% (binomial series 0.077712).
- 6% convertible quarterly = 6.136%; half-yearly 6.09%; monthly 6.168%.

TRAP

- Nominal is not the actual interest earned in a year; effective is higher for m above 1.
- Compare offers by effective rates.

1.6 Discount

- Discount = amount due minus its present value. Rate of discount d = discount on 1 due a year hence.

Link	Formula
d and v	d = 1 - v; v = 1 - d = 1/(1+i)
d and i	d = i v = i/(1+i)
i and d	i = d/v = d/(1-d)
Product	i - d = i d
PV by discount	v^n = (1-d)^n

- i = interest on 1 for a year = discount on (1+i) due a year hence = accumulated value of d after a year.
- d = discount on 1 due a year hence = interest for a year on v = present value of i due a year hence.

FORMULA AND WORKED SUM

- **i = 0.06:** d = 0.06/1.06 = 0.0566. d = 0.08: i = 0.08/0.92 = 0.08696.
- **Rs 1,000 due 10 yrs at d = 5%:** 1,000 x 0.95^10 = about Rs 598.7.
- **Rs 2,000 due 5 yrs at d = 4%:** 2,000 x 0.96^5 = Rs 1,630.75.

TRAP

- i is reckoned on the present sum; d on the sum due later.
- d is the PV of i; i is the accumulated value of d.

1.7 Varying rates

- Work in stages; multiply: (1+i)^m x (1+i')^n x (1+i'')^p.
- Periods may be years, half-years, quarters; rate must match its period; units may differ by stage.

FORMULA AND WORKED SUM

- **Rs 2,000:** 5% for 7 yrs; 5% half-yearly for 3 yrs (6 half-years at 2 1/2%); 6% quarterly for 5 yrs (20 quarters at 1 1/2%) = 2,000 x 1.40710 x 1.15969 x 1.34686 = Rs 4,395.61.
- PV of Rs 500 due 10 yrs; 5% for 4 yrs, 6% half-yearly for 6 yrs: 500 x 0.82270 x 0.70138 = Rs 288.51.
- Rs 1,000 at 5% 2 yrs then 6% 3 yrs = Rs 1,313.10.

TRAP

Multiply stages, never add.

CHAPTER 1 (CONTD.)

Varying rates, tables, equation of value

1.1 Values not in the tables
1.2 Time and rate from tables
1.3 Equation of value
1.4 Equated time of payment
1.5 Worked equation-of-value problems
1.6 Practice results
1.7 Binomial theorem

MUST KNOW

1. **Power beyond the table:** split it, v^62 = v^50 x v^12. Between table values: interpolate.
2. **Fractional period, more accurate:** multiply by (1 + i(4)/4) for a quarter year.
3. **Find n:** interpolate along a rate column. Find i: interpolate along a row of n.
4. **Equation of value:** payments = benefits at one date; any date, same answer.
5. Earlier payments are accumulated; later payments are discounted.
6. Equated time approx = sum (payment x time) / total payment.
7. **Binomial:** (1+x)^n has n+1 terms for whole n; other n need x numerically below 1.

1.1 Values not in the tables

Difficulty	Example
Fractional rate or period	(1.052)^28; 17 yrs **3 months**
Period not tabulated	(1.06)^72 (table to n = 50)
Table read backwards	find n or i from a given value

FORMULA AND WORKED SUM

- **Ex 26:** (1.09)^17.25: 4.32763 + 0.38949/4 = 4.42500 (interpolation)
- **More accurate:** 4.32763 x (1 + i(4)/4) = 4.32763 x 1.021778 = 4.42188; i(4) = 0.087113

- **(1+i)^(n+k) = (1+i)^n x (1+ki):** same as simple interpolation (4.42500).

FORMULA AND WORKED SUM

- **Ex 27:** v^62 at 4% = v^50 x v^12 = 0.14071 x 0.62460 = 0.08789
- **Ex 28:** (1.03)^58.4 = 5.55341 + 0.4 x 0.16658 = 5.62004 (exact power 5.61945)

TRAP

Interpolation (4.42500) differs from the (1 + i(4)/4) method (4.42188).

1.2 Time and rate from tables

FORMULA AND WORKED SUM

- **Ex 29:** (1.04)^n = 2; n=17: 1.94790, n=18: 2.02582; 0.05210/0.07792 = 0.67; n = 17.67 yrs
- **Ex 30:** 1000 (1+i)^15 = 1750; 3.5%: 1.67535; 4%: 1.80094; 0.07465/0.12559 x 0.005 = 0.00297; i = 3.797% p.a.

1.3 Equation of value

- Value t years before due date = C v^t (less than C); n years after = C (1+i)^n (more than C).
- Reduce payments at different times to one amount at one chosen date.
- Payments made = benefits received (no profit, no loss) at the same date.
- Date chosen does not matter; a convenient date saves arithmetic.

TRAP

Do not reverse: earlier payments accumulate, later payments discount.

1.4 Equated time of payment

- One payment equal to the total, at the time that makes it equivalent.

FORMULA AND WORKED SUM

- Exact (Rs 100 at 2, Rs 200 at 4, Rs 400 at 6 yrs, 4%): 100v^2 + 200v^4 + 400v^6 = 700 v^n; 579.540/700 = 0.82791; n = 4.82 yrs
- **Approx:** (100x2 + 200x4 + 400x6)/700 = 4.857 yrs (a little higher; favours the party paying the single sum)

TRAP

Divide by the total of the payments, not the number of payments.

1.5 Worked equation-of-value problems

FORMULA AND WORKED SUM

- **Ex 31:** Rs 1,000 now = P at 3, 6, 10 yrs, 6%: v^3 .83962 + v^6 .70496 + v^10 .55839 = 2.10297; P = Rs 475.52
- **Same at 10 yrs:** 1000 x 1.06^10 = 1790.85; (1.50363 + 1.26248 + 1) P = 3.76611 P; P = Rs 475.52
- **Ex 32:** loan 2000, 4% p.a. half-yearly (2%); repay 400 at 2 yrs, 600 at 4 yrs; final at 7 yrs: 2000 x 1.02^14 = 2638.96; 400 x 1.02^10 = 487.596; 600 x 1.02^6 = 675.696; P = Rs 1,475.67

- Choose the final-payment date so the unknown has no multiplying factor.

FORMULA AND WORKED SUM

- **Ex 33:** bond 100, 4% yearly, 3 yrs, par; yield 5%: 4 (.95238 + .90703 + .86384) + 100 x .86384 = Rs 97.28 (below par)

1.6 Practice results

- **Rs 300 at 7%, 12 yrs:** 675.66 (interest 375.66). PV of 1000 in 50 yrs at 5%: 87.20.

- **512 (1+i)^3 = 719.32:** i = **12%**. **Rs 2,809** after 2 yrs at **6%**: principal 2,500. **Rs 8,000**: CI 820 (2 yrs), 1,261 (3 yrs): **5%**.
- **Endowment Rs 20,000 paid 12 yrs early: 6%** 9,939.39; **7%** 8,880.24; **8%** 7,942.28.
- **Rs 1,000, 10 yrs, 12% convertible quarterly:** 3,262.04; **3%** per half-year: 1,806.11.
- **PV of 1,000 in 10 yrs: 6%** monthly 549.63; **1%** per quarter 671.65.
- Effective **6%** = nominal **5.8695%** convertible quarterly. **12%** convertible monthly = **12.6825%** effective.
- NSC 1,000 to 1,901 in 6 yrs = **11.30%**; FD 1,000 to 1,629 in 5 yrs = **10.25%**; NSC higher.
- **Doubling in 10 yrs half-yearly: 3.5265%** per half-year, nominal **7.05%**.

1.7 Binomial theorem

- (x+y)^2 = x^2 + 2xy + y^2; (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.

FORMULA AND WORKED SUM

- (x+y)^4 coefficients 1, 4, 6, 4, 1; (x+y)^5 coefficients 1, 5, 10, 10, 5, 1.

- **Coefficients:** n; n(n-1)/1.2; n(n-1)(n-2)/1.2.3 ...; x falls by 1, y rises by 1 each term.

FORMULA AND WORKED SUM

- (1+x)^n = 1 + nx + n(n-1)/1.2 x^2 + ...; n! = 1.2.3...n

- **Whole n:** n+1 terms. Other n: infinite terms, valid only if x is numerically less than 1.
- (a+b)^n = a^n (1 + b/a)^n = k(1+x)^n with k = a^n, x = b/a, a numerically larger.